Multimode surface-wave analysis of near-surface data.

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1 Abstract

The near-surface shear-velocity profile can be determined from the inversion of surface-wave dispersion data. Several methods of analysis have been developed in the past, mostly using the phase information of seismic signals observed on linear arrays of sensors. One drawback of these techniques is a possible misidentification between modes and secondary lobes resulting from the limited appearture of the array. Another limitation comes from the contamination by random and/or signal generating noise since the whole wavefield is taken into account. In this work, we propose to overcome these limitations by adding a group-delay time information in the analysis. The wavefield is mapped into the group-velocity/phase-velocity (U-c) domain which enables us to improve the dispersion measurements. The observed multimode dispersion data are then inverted in terms of a 1D velocity profile. We show both on synthetics and field data that the U-c diagrams greatly facilitate the identification of each modes and that our inversion procedure quickly converge to the expected models.

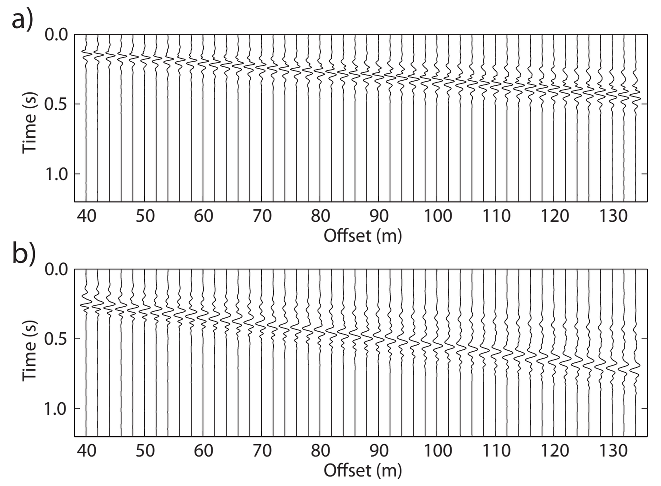

Figure 2: *Synthetic seismograms computed by summing the first six Rayleigh modes (Herrmann, 2006). (a) and (b) correspond to models A and B shown on* **Figure 5**.

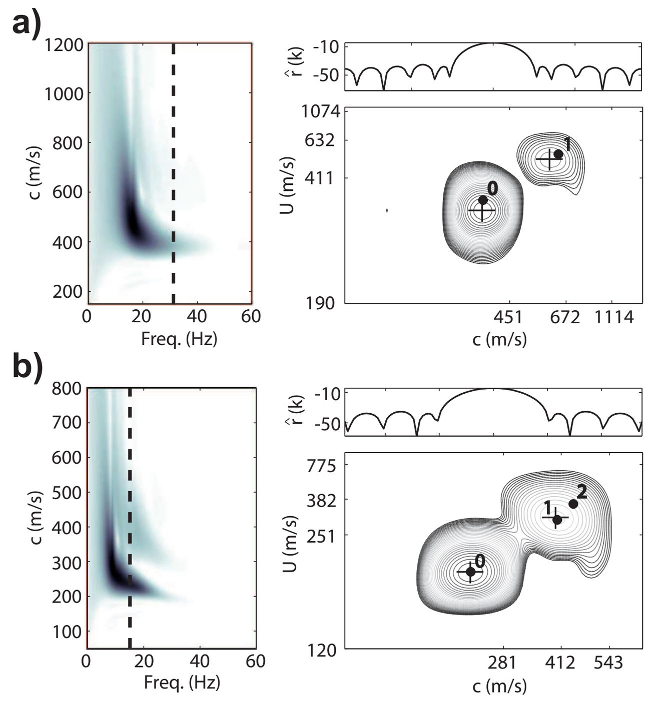

Figure 3: *f-c Diagrams (left) and U-c Diagrams (right) corresponding to the synthetics on* **Figure 2**. *Dotted black lines: frequencies used to calculate the U-c diagrams. Black dots: theoretical values. Crosses: location of peak maxima.*

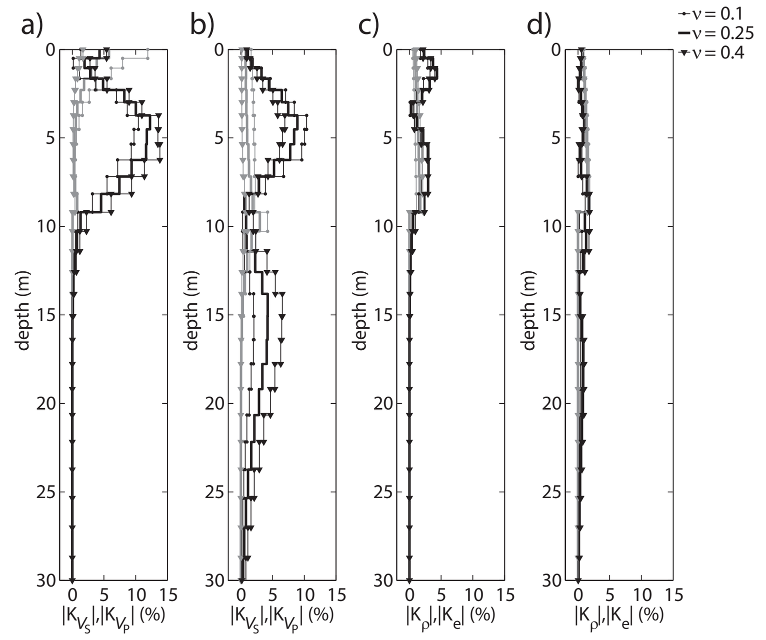

Figure 1: *Kernel sensitivity of phase-velocities at 28Hz computed for model A (**Figure 5**a-b). K_{V_P}, K_e (grey lines), K_{V_S} and K_ρ (dark lines) are displayed for (a) the fundamental mode and (b) the first overtone for different Poisson ratios ν ranging from 0.1 to 0.4.*

2 Dispersion measurements : U-c Diagrams

Multi-mode signals recorded by an array of N sensors are stacked in order to reinforce the individual modes by constructive interference :

$$G_{\omega_0}(k,\omega) = \sum_{n=0}^{N-1} w_n S_{\omega_0}(x_n,\omega)\exp[-\mathrm{i}K(k,\omega)x_n],$$

where $S_{\omega_0}(x_n,\omega)$ is the time Fourier transform of a record observed at epicentral distance x_n and filtered around the circular frequency ω_0. The last term in this expression is the phase-shift filter with $K(k,\omega)$ defined as : $K(k,\omega) = k + \frac{\omega-\omega_0}{U_c}$ where k is the wavenumber and U_c is the central group-velocity of the multi-mode wave packet.

We call "U-c diagram" the modulus of the inverse Fourier transform $g_{\omega_0}(k,t)$ of $G_{\omega_0}(k,\omega)$. It will exhibit peaks at group group-velocity $U_m(\omega_0)$ and phase-velocity $c_m(\omega_0)$, related to each surface-wave mode m (*Cara* 1976, *Duputel et al.* 2009).

The series of diagrams plotted at different frequencies ω_0 then allows the analyst to retrieve the fundamental and the higher-mode dispersion curves.

3 Inversion for 1-D velocity profile

We consider a thin layered model of the subsurface where we invert for V_S only since V_P and ρ have a small influence on the phase-velocities (**Figure 1**). This is a non-linear problem which is solved using a two-step inversion scheme (*Duputel et al.*, 2009). The first step is a pre-inversion step providing the a priori model which is necessary to run the second step based on a quasi-Newton algorithm. Two alternative approaches were followed to choose the a priori information on the model parameters: 1) An empirical conversion of the fundamental mode phase velocity into V_s and 2) extraction of an optimum a priori information from a model-library made of 5000 models.

4 Synthetic tests

The fundamental mode and the first higher mode are clearly visible on the U-c diagrams (**Figure 3**). On **Figure 4**, the fundamental mode phase-velocity measurements fit well with the theoretical values, even for frequencies lower than 15Hz where the classical f-k method fails. On **Figure 4**a, we note a phase-velocity misfit which is greater for the first overtone than for the fundamental mode. This is probably due to the low amplitude of the first overtone which is clearly visible on the U-c diagram (**Figure 3**a(right)), but merely detectable on the f-c diagram (**Figure 3**a(left)).

Figure 5 depicts the inversion results corresponding to the dispersion data measured on U-c Diagrams.

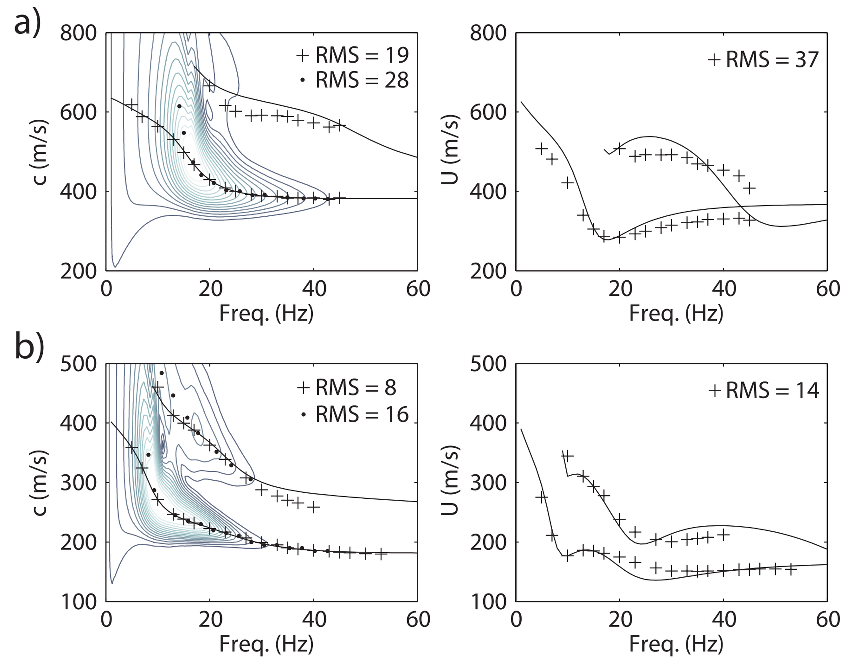

Figure 4: *Measured and Theoretical phase-velocities c and group-velocities U for model A (a) and model B (b) (**Figure 5**). Amplitude contourmap of f-c diagrams are displayed. Solid lines: theoretical values, black crosses: values measured on the U-c diagrams, black dots: phase-velocities measured on the f-c diagrams.*

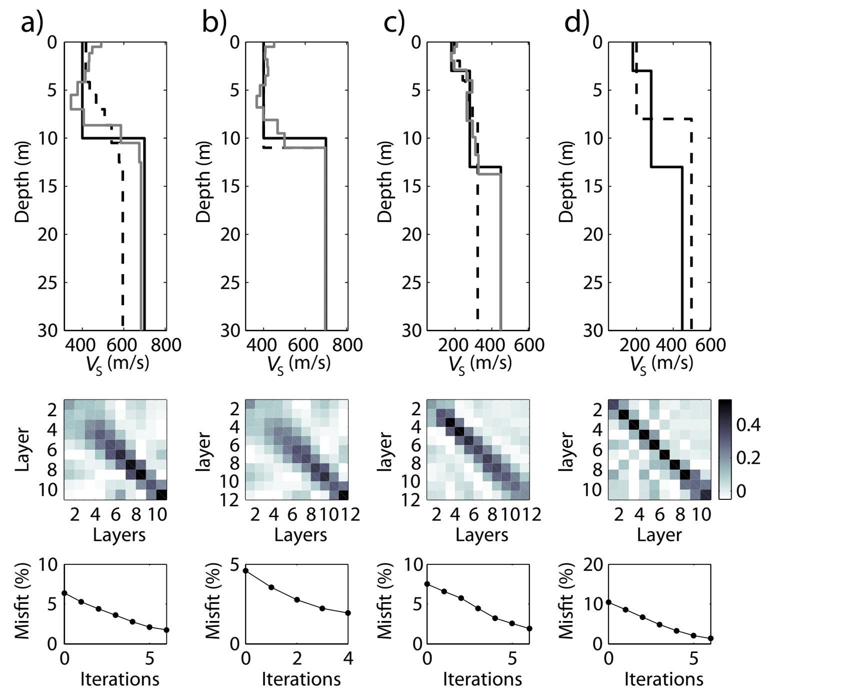

Figure 5: *Inversion results from measurements on synthetic U-c diagrams for the model A (a-b) and the model B (c-d). Resolution matrices and data misfits are shown. (a) and (c): a priori empirical conversion of c into V_s. (b) and (d): use of a model-library. Plain black line: exact model. Doted black line: a priori model. Grey line: inverted model.*

5 Application to field data

Dispersion curves related to the records displayed in **Figure 6** are estimated by using the U-c diagram technique. Note on **Figure 7** that only the fundamental mode and the first overtone are well excited.

Even if the two initial models differ, the final inverted models are remarkably similar (**Figure 8**) and a good fit is observed between observed and predicted phase-velocities (**Figure 9**). This emphasizes the robustness of this solution that is confirmed by the resolution matrices which show a good overall resolution.

Three sets of layers can clearly be distinguished on **Figure 8**. They correspond to the local soil structure which is well known from previous seismic refraction investigations (e.g. *Bano et al.*, 2002). A geological interpretation of is given in Table 1.

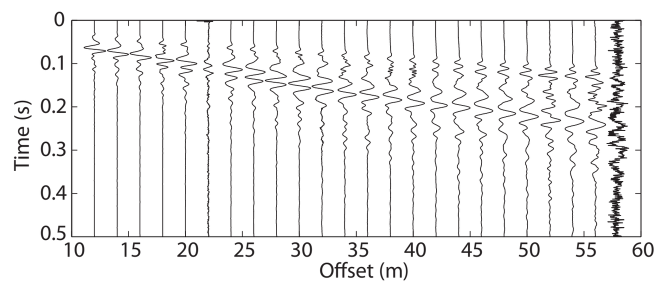

Figure 6: *The shot gather obtained in Riedseltz, Alsace (France), by using a sledgehammer as the energy source.*

$e(m)$	$\langle V_s\rangle\,(m/s)$	lithology
2.20	178	Undifferentiated loess (Wurm age)
14.14	330	Pliocene sands
∞	507	Oligocene marls

Table 1: *Interpretation of the inverted shear velocity profiles displayed on **Figure 8**. e: layer thickness, $\langle V_s\rangle$: averaged shear-velocity in each layer. The lithology can be directly observed on a quarry which is next to the seismic profile.*

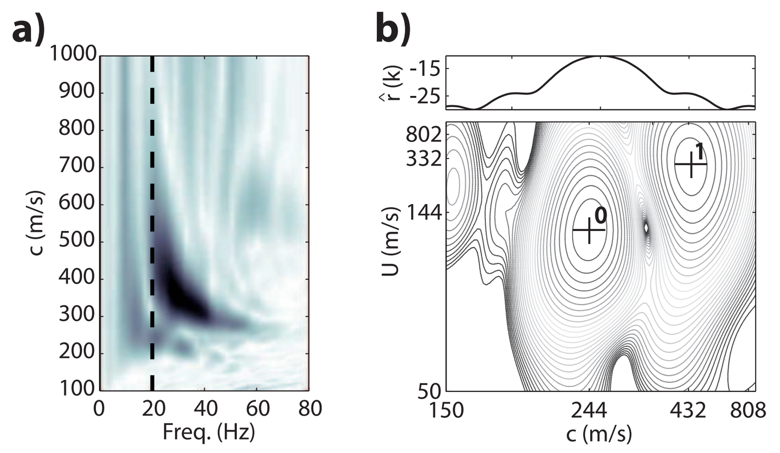

Figure 7: *f-c Diagram (left) and U-c Diagram (right) obtained by stacking the records shown in **Figure 6**. Dotted black line: frequency used to calculate the U-c diagram (20Hz). See caption of **Figure 3**.*

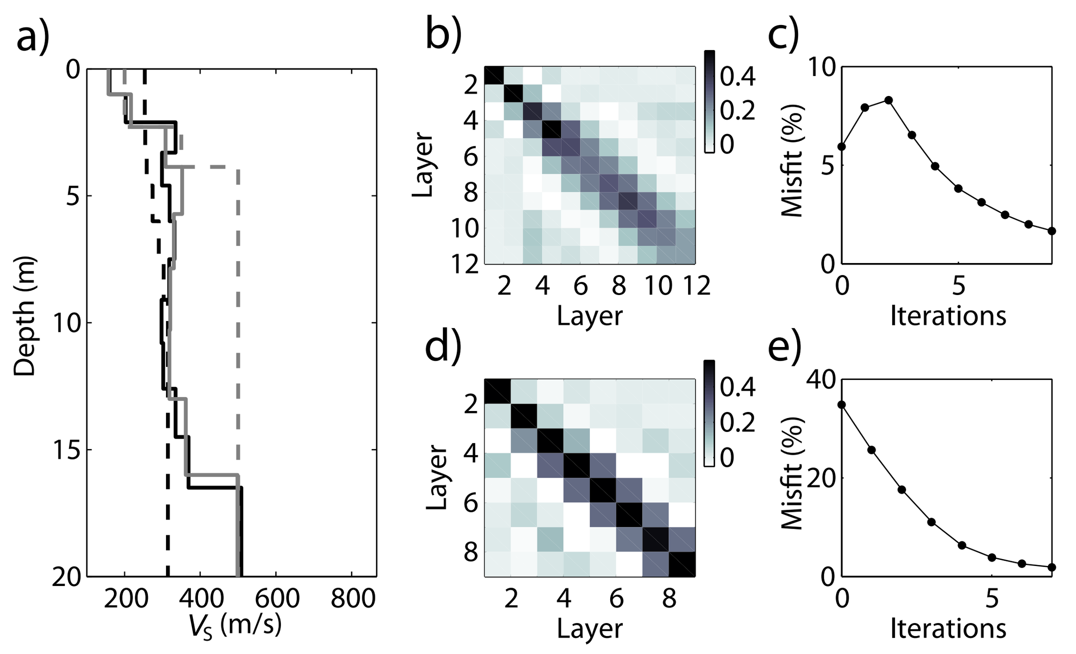

Figure 8: *Inversion results from the phase-velocities measured on synthetic U-c diagrams. In (a), plain lines: inverted models, dotted lines: the a priori models. Black lines correspond to inversions based on the a priori conversion of c into V_s associed with the resolution matrix in (b) and the data missfit in (c). Grey lines correspond to the use of a model-library (the same as for **Figure 5**). The corresponding resolution matrix is shown in (d) and data missfit in (e).*

6 Conclusions

We show that the discrimination between the fundamental and the first higher mode is clearly made easier by using U-c diagrams instead of classical f-k analysis. The high quality dispersion data thus obtained can be processed to infer the near-surface shear velocity structure with a better depth resolution than when using fundamental mode dispersion only.

Our simple inversion procedure converges rapidly to acceptable solutions both for synthetic and actual data. The fact that the inverted V_s profiles depend weakly on the a priori model gives us confidence in the robustness of the solutions.

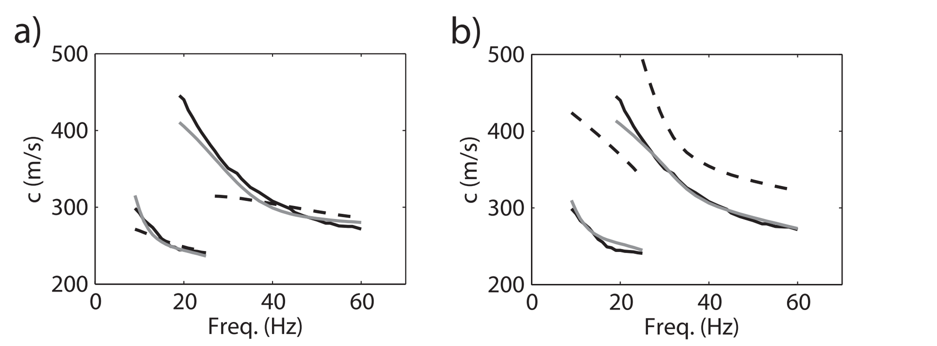

Figure 9: *Plain black line: measured phase-velocities. Doted black line: data predicted from the a priori model. Grey line: data predicted from the inverted model. (a) A priori conversion of c into V_s. (b), Use of the model-library.*

The source-receiver configuration and the test area which we consider in this paper corresponds to a cheap, classical experiment in near-surface prospecting. This limits the higher-mode content in the observed surface-wave signals. A simple but more expensive configuration to increase the resolution of the method at depth could, for example, consist of using a buried instead of a surficial source.

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References

Bano, M., J. Edel, G. Herquel, and EPGS class 2001-2002, Geophysical investigation of a recent shallow fault, *The Leading Edge*, *21*(7), 648–650, 2002.

Cara, M., Observation d'ondes S_a de type SH, *Pure and Applied Geophysics*, *114*, 141–157, 1976.

Duputel, Z., M. Cara, L. Rivera, and G. Herquel, Improving the analysis and inversion of multimode Rayleigh-wave dispersion by using group-delay time information observed on arrays of high frequency sensors, *Accepted in Geophysics*, 2009.

Herrmann, R. B., *Computer Programs in Seismology*, Saint Louis University., 2006.